PHYSICS LAB MANUAL

PHYS101

Science:

If you don't make mistakes,
you're doing it wrong.

If you don't correct those mistakes,
you're doing it really wrong.

If you can't accept that you're mistaken,
you're not doing it at all.
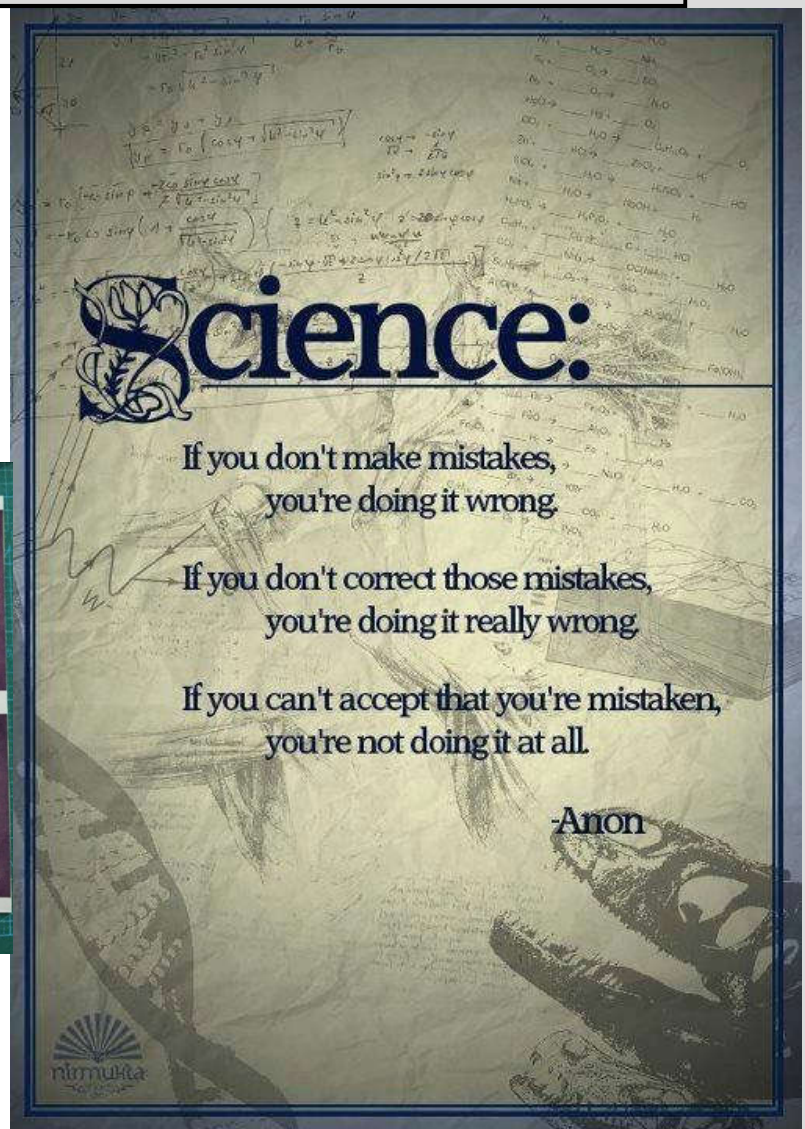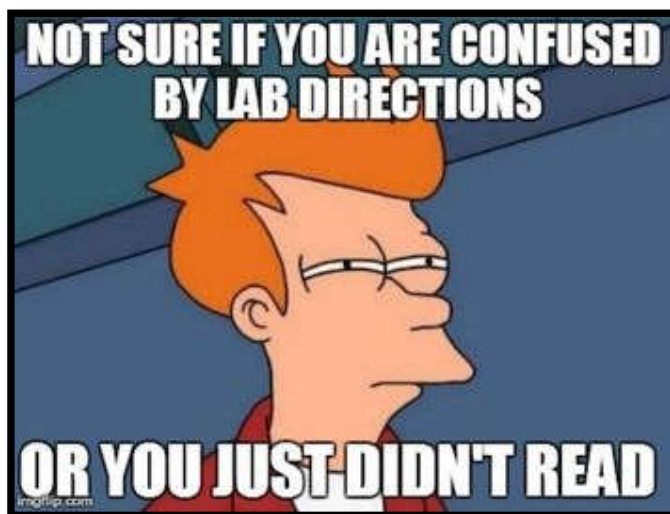
-Anon

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Introduction

You are **highly recommended** to read this manual very carefully!

1.1 Sources of Error in Measurement

1.1.1 Does Every Measurement Have an Error?

It is noteworthy to mention that not every measurement has an error. Consider the case when you wish to determine the number of students in the class. This can easily be achieved via counting, without any error. However, if we wish to determine the number of oxygen molecules in the room, it is no longer possible to count them. In this case, the number can be estimated with an uncertainty. This uncertainty determines the measurement accuracy.

1.1.2 Uncertainty versus Error

The concept **uncertainty** plays a vital rule during practical sessions. Thus, you must make sure that you understand it well! For this reason, let us start with a simple example. Imagine you are asked to take a measurement via using a meter ruler. Once you take your data, you must be confident about **how to record it**.

Assume that you have measured the length to be 25.1 cm. Even though you may 'somehow' convince yourself that this is one true measurement, you cannot claim that it is 100% accurate.

You must be aware of the terms **uncertainty** and **error**, alongside with their differences. They may sound like the same thing, but they actually are not! What makes this discussion even more interesting is once the word **mistake** is involved within the subject.

Your teaching assistants have been hearing these kind of questions very frequently during the lab sessions: "I have done no mistakes today, but my percentage error has a non-zero value. I am sure I have done everything right!!! So is this logical?" The answer is no!! It indeed has no logical sense. Errors can be thought as a concept that has to exist whenever you are taking a measurement, no matter whether you wish it to be there or not, whereas the mistakes are dependent on the individual himself. You are able to control the mistakes; however, the errors are unavoidable!

And what about the **uncertainty**? Uncertainty stands for the **methodology of manifesting errors**. If you can manage to minimize the errors, it will in turn have an effect on the uncertainty. As we have already pointed out, errors should not be considered as mistakes. Yet, they must exist! Reading the list below may help you to understand why it is so.

There are a variety of reasons why errors arise. Some of these are listed below.

Human Error
Whilst taking a measurement, you may all claim you have observed something slightly different. You are all carrying out the experiments as good as you can, or scientifically speaking, up to a certain level of accuracy. The measuring techniques may vary from observer to observer. Therefore, to sum up, this type of error may occur because of being unexperienced or choosing a less accurate way of taking the concerned measurement.

Instrumental Limitations on the Devices
There exist calibrations of finite width. Each device has its own limitation. It is not possible to measure the length of a substance and record it as 5.16472847294728947204 cm, by using a ruler.

Errors Due to External Influences

If your system is not isolated well while you are carrying out an experiment to find specific heat capacity of a substance, some heat will be lost to the surroundings and this will make your reading less accurate. This is an example of an external influence.

Parallax Error

Just by examining the meaning of the word parallax, we can understand how this error arises. Parallax means alteration, hence the observations will be dependent on the angle of observation. This error can be minimized via experience.
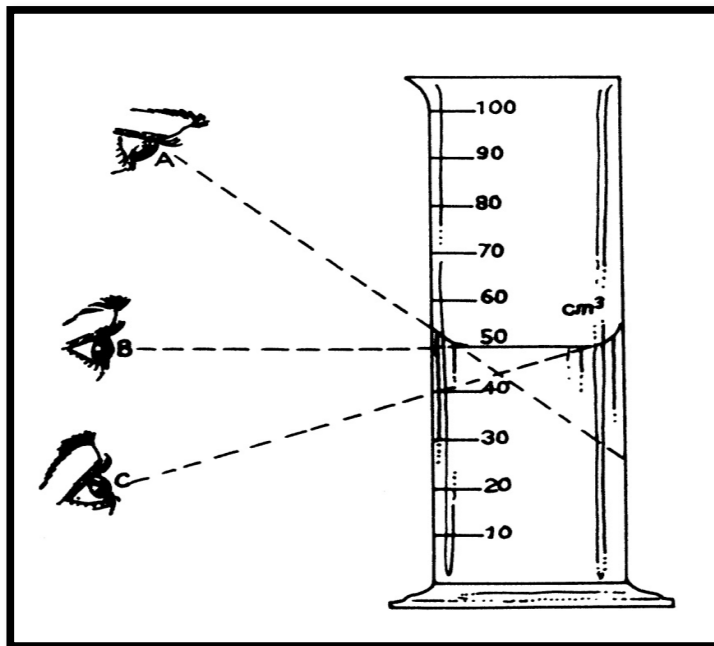
The correct position to take the measurement in the figure above is position B. Try to take your measurements perpendicular to the observation point.

1.2 Significant Figures, Rounding, and Uncertainty

1.2.1 Significant Figures

Basic Rule: Non-zero digits are always significant. Any zeros except for at the beginning are significant.

Some Examples

$$26.380 \rightarrow \text{ 5 significant figures}$$
$$7.94 \rightarrow \text{ 3 significant figures}$$
$$0.00980 \rightarrow \text{ 3 significant figures}$$

The last number can be stated in scientific notation as 9.80×10^{-3}.

1.2.2 Rounding Numbers

To keep the correct number of significant figures, numbers must be rounded off. The discarded digit is called the remainder. There are three rules for rounding:

Rule 1: If the remainder is less than 5, drop the last digit.
Rounding to one decimal place: 5.346→5.3

Rule 2: If the remainder is greater than 5, increase the final digit by 1.
Rounding to one decimal place: 5.798→5.8

Rule 3: If the remainder is exactly 5 then round the last digit to the closest even number. This is to prevent rounding bias. Remainders from 1 to 5 are rounded down half the time and remainders from 6 to 10 are rounded up the other half.
Rounding to one decimal place: 3.55→3.6, also 3.65→3.6.

1.2.3 Uncertainty in Measurement

Every measurement apparatus has a limitation in precision. A Voltmeter with 4 digits cannot measure up to a precision of 5 digits. Other factors, as previously explained, may as well influence the measurement. The uncertainty of a quantity x is denoted by Δx

Every measurement reading is written together with its uncertainty in the following form.

$$(x \pm \Delta x)\ unit$$

Absolute and Relative Uncertainty

Absolute uncertainty: $3.52 \pm 0.03\ mm$.

Relative uncertainty: $3.52\ mm \pm 1\%$. Note that relative uncertainty was found via evaluating $0.03/3.52 \times 100\ \%$.

Important Remark: You are expected to display **uncertainties to two significant figures** during the lab sessions. Having done so, you must adjust your result in such a way to have the same number of decimal places as the uncertainty.

Let us give you an example on how to follow this procedure.

Example
You have taken a measurement for a quantity together with its uncertainty and wrote that

$$x = 3.2 \ cm$$
$$\Delta x = 0.05 \ cm.$$

How should you record this measurement on your spreadsheet?

Step#1: Rewrite the uncertainty to two significant figures, i.e. $\Delta x = 0.050 \ cm$.

Step#2: Quote your result in a way that its decimal places match the one for the uncertainty, i.e. $x = 3.200 \ cm$.

Step#3: Finalize your answer as follows.

$$x = (3.200 \pm 0.050) \ cm$$

1.2.4 Practice Questions

Rewrite the following measurements and their uncertainties in the format you are taught throughout this manual.

1) $x = 5.0$ cm, $\Delta x = 0.01 \ cm$.

2) $g = 10.42 \ m/s^2$, $\Delta g = 1.80 \frac{m}{s^2}$.

3) $v = 42.30 \ m/s$, $\Delta v = 4.3 \frac{m}{s}$.

1.3 Some Basic Statistics

1.3.1 Mean

Example
If you have repeated a measurement five times and have observed them as 5 cm, 6 cm, 5 cm, 8 cm and 6 cm respectively, the most precise way of representing your measured value is finding the average value, i.e. the mean value. This can be achieved as follows.

$$\bar{x} = \frac{5 + 6 + 5 + 8 + 6}{5} = 6$$

Increasing the number of measurements always provides us with a higher accuracy and our final result in turn becomes more reliable.

1.3.2 Variance
The variance represents a measure of how the data distributes itself about the mean or the expected value. While evaluating the variance, there is an important point to be considered.

$$v_P = \sum \frac{(x_m - \bar{x})^2}{n}$$

$$v_s = \sum \frac{(x_m - \bar{x})^2}{n - 1}$$

where v_p and v_s represent population and sample variance, respectively.

1.3.3 Gaussian Distribution
If only random errors affect a measurement, it can be shown mathematically that in the limit of an infinite number of measurements, the distribution of values follows a normal distribution (i.e. the bell curve or the Gaussian distribution at the below). This distribution has a peak at the mean value x_{avg} and a width given by the standard deviation σ.
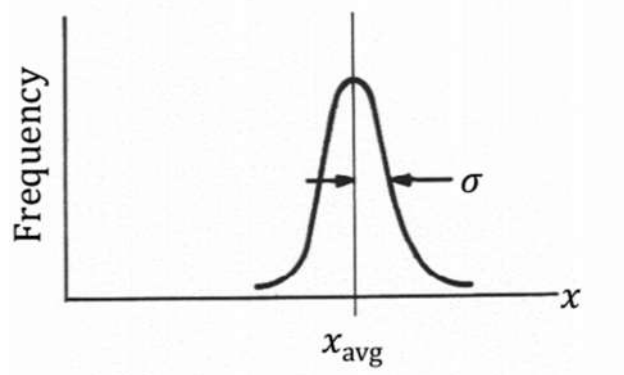
Obviously, we never take an infinite number of measurements. However, for a large number of measurements, say, N~10 or more, measurements may be approximately normally distributed. In that event we use the formulae below:

Mean (x_{avg})	The average of all values of x (the "best" value of x)	$x_{avg} = \dfrac{x_1 + x_2 + \cdots + x_N}{N}$
Range (R)	The "spread" of the data set. This is the difference between the maximum and minimum values of x	$R = x_{max} - x_{min}$
Uncertainty in a measurement (Δx)	Uncertainty in a single measurement of x. You determine this uncertainty by making multiple measurements. You know from your data that x lies somewhere between x_{max} and x_{min}.	$\Delta x = \dfrac{R}{2} = \dfrac{x_{max} - x_{min}}{2}$
Uncertainty in the Mean (Δx_{avg})	Uncertainty in the mean value of x. The actual value of x will be somewhere in a neighborhood around x_{avg}. This neighborhood of values is the uncertainty in the mean.	$\Delta x_{avg} = \dfrac{\Delta x}{\sqrt{N}} = \dfrac{R}{2\sqrt{N}}$
Measured Value (x_m)	The final reported value of a measurement of x contains both the average value and the uncertainty in the mean.	$x_m = x_{avg} \pm \Delta x_{avg}$

Most of the time we will be using the formulae for small data sets. However, occasionally we perform experiments with enough data to compute a meaningful standard deviation. In those cases, one can take advantage of some computer packages for computing x_{avg} and σ.

1.4 Propagation of Uncertainties[1]

Oftentimes we combine multiple values, each of which has an uncertainty, into a single equation. In fact, we do it when we measure something with a ruler. Take, for example,

$x = 1.0\text{cm} \pm 0.05\text{cm}$ $y = 4.63\text{cm} \pm 0.05\text{cm}$

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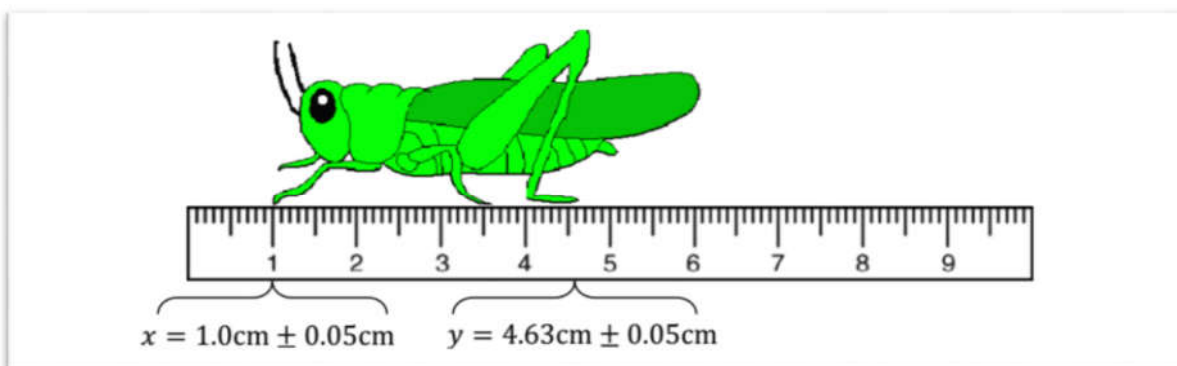
[1] † *Reference for the two tables in this section:* University of Pennsylvania, Department of Physics & Astronomy, Lab Manual for Undergraduates.
Note that throughout the second table provided from this source, the uncertainties are not quoted to 2 significant figures. However, this will not be the case for us! These examples are only provided for you to understand how to work on error propagation.

measuring the distance from a grasshopper's front legs to his hind legs. For rulers, we will assume that the uncertainty in all measurements is one-half of the smallest spacing.

The measured distance is $d_m = d \pm \Delta d$ where $d = 4.63\text{cm} - 1.0\text{cm} = 3.63\text{cm}$. What is the uncertainty in d_m? You might think that it is the sum of the uncertainties in x and y; namely:

$\Delta d = \Delta x + \Delta y = 0.1\text{cm}\cdot$

However, statistics tells us that if the uncertainties are independent of one another, the uncertainty in a sum or difference of two numbers is obtained by quadrature:

$\Delta d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = 0.07\text{cm}.$

The way these uncertainties combine depends on how the measured quantity is related to each value. Rules for how uncertainties propagate are given below.

Addition/Subtraction	$z = x \pm y$	$\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
Multiplication	$z = xy$	$\Delta z = \|xy\| \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$
Division	$z = \dfrac{x}{y}$	$\Delta z = \left\|\frac{x}{y}\right\| \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$
Power	$z = x^n$	$\Delta z = \|n\|x^{n-1}\Delta x$
Multiplication by a Constant	$z = cx$	$\Delta z = \|c\|\Delta x$
Function	$z = f(x,y)$	$\Delta z = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\Delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\Delta y)^2}$

Examples

Addition

The sides of a fence are measured with a tape measure to be 124.2cm, 222.5cm, 151.1cm and 164.2cm. Each measurement has an uncertainty of 0.07cm. Calculate the measured perimeter P_m including its uncertainty.

$$P = 124.2\text{cm} + 222.5\text{cm} + 151.1\text{cm} + 164.2\text{cm} = 662.0\text{cm}$$

$$\Delta P = \sqrt{(0.07\text{cm})^2 + (0.07\text{cm})^2 + (0.07\text{cm})^2 + (0.07\text{cm})^2} = 0.14\text{cm}$$

$$\boxed{P_m = 662.0 \pm 0.1\text{cm}}$$

Multiplication

The sides of a rectangle are measured to be 15.3cm and 9.6cm. Each length has an uncertainty of 0.07cm. Calculate the measured area of the rectangle A_m including its uncertainty.

$$A = 15.3\text{cm} \times 9.6\text{cm} = 146.88cm^2$$

$$\Delta A = 15.3\text{cm} \times 9.6\text{cm} \sqrt{\left(\frac{0.07}{15.3}\right)^2 + \left(\frac{0.07}{9.6}\right)^2} = 1.3\text{cm}^2$$

$$\boxed{A_m = 147 \pm 1\text{cm}^2}$$

Power/Multiplication by Constant

A ball drops from rest from an unknown height h. The time it takes for the ball to hit the ground is measured to be $t = 1.3 \pm 0.2s$. The height is related to this time by the equation $h = \frac{1}{2}gt^2$ where $g = 9.81\text{m/s}^2$. Assume that the value for g carries no uncertainty and calculate the height h including its uncertainty.

$$h = \frac{1}{2}(9.8\text{m/s}^2)(1.3s)^2 \approx 8.281\text{m}$$

$$\Delta h = \frac{1}{2}(9.8\text{m/s}^2)(2 \times 1.3s \times 0.2s) \approx 2.5\text{m}$$

$$\boxed{h_m = 8 \pm 3\text{m}}$$

1.5   The Least Squares Line

There are two possible approaches to determine the least squares line. Either we consider the square of the vertical distance between the measured points and the least square line or we consider the square perpendicular distance between the measured point and the least square line.

vertical offsets   **perpendicular offsets**
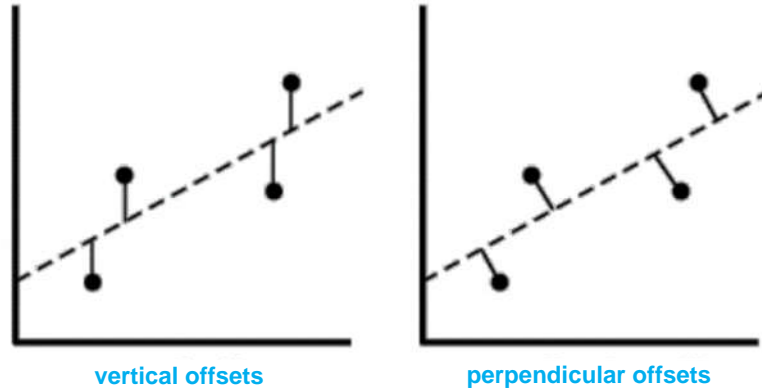
In this manual, we mainly focus on the sum of the squares of the vertical distances between the best fit line. It is given by

$$S(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} (y_i - (mx_i + c))^2,$$

where $y = mx + c$ denotes the best fit line. The problem of finding the coefficients m and c is rather straight forward. We have to minimize the sum of the squares $S(\mathbf{x}, \mathbf{y})$ with respect to the coefficients m and c, using the straightforward method we have learned in the course of Calculus. To this end, we take the derivative of $S(\mathbf{x}, \mathbf{y})$ with respect to m and c, and in sequel equate them to 0. This results in

$$\frac{\partial S}{\partial m} = \sum_{i=1}^{n} 2(y_i - mx_i - c)(-x_i) = 0,$$

$$\frac{\partial S}{\partial c} = \sum_{i=1}^{n} 2(y_i - mx_i - c) = 0.$$

Now, one must solve the above equations for m and c. After dividing the both equations by 2, we can rewrite the equations as following

$$-\sum_{i=1}^{n} x_i y_i + m \sum_{i=1}^{n} x_i^2 + c \sum_{i=1}^{n} x_i = 0,$$

$$\sum_{i=1}^{n} y_i - m \sum_{i=1}^{n} x_i - cn = 0.$$

The above equations can be decoupled in one equation as follows:

$$\begin{pmatrix} -\sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i \end{pmatrix} + a\begin{pmatrix} \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} x_i \end{pmatrix} + b\begin{pmatrix} \sum_{i=1}^{n} x_i \\ n \end{pmatrix} = 0,$$

or

$$\begin{pmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & n \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} x_i y_i \\ -\sum_{i=1}^{n} y_i \end{pmatrix}.$$

The solution of the above equation can be obtained by using the Kramer's rule:

$$\begin{pmatrix} m \\ c \end{pmatrix} = \frac{1}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}\begin{pmatrix} n & -\sum_{i=1}^{n} x_i \\ -\sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{pmatrix}\begin{pmatrix} \sum_{i=1}^{n} x_i y_i \\ -\sum_{i=1}^{n} y_i \end{pmatrix},$$

which finally gives the deserved results for the slope m and intercept c as follows:

$$m = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2},$$

$$c = \frac{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}.$$

Experiment 1: Force Table
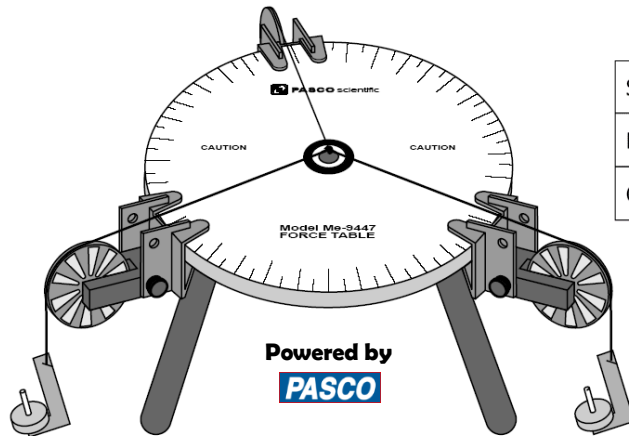
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EQUIPMENT NEEDED: Force Table, 3 pulleys and pulley clamps, 3 mass hangers, mass set, string, metric ruler, protractor, 2 sheets of paper.

1- PROPOSE

The purpose of this experiment is to use the force table to experimentally determine the force which balances two other forces. This result is checked by adding the two forces by using their components.

2- THEORY

This experiment finds the resultant of adding two vectors by two methods: experimentally and by components. (Some general details on adding vectors, specifically forces, will be given by the lab assistant).

Note that in all cases, the force caused by the mass hanging over the pulley is found by multiplying the mass by the acceleration due to gravity.

A- Experimental Method

Two forces are applied on the force table by hanging masses over pulleys positioned at certain angles. Then the angle and mass hung over a third pulley are adjusted until it balances the other two forces. This third force is called the equilibrant (F_E) since it is the force which establishes equilibrium. The equilibrant is not the same as the resultant (F_R). The resultant is the addition of the two forces. While the equilibrant is equal in magnitude to the resultant, it is in the opposite direction because it balances the resultant (see Figure 1). So, the equilibrant is the negative of the resultant:
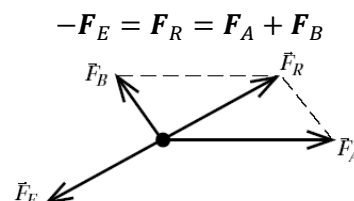
$$-\boldsymbol{F}_E = \boldsymbol{F}_R = \boldsymbol{F}_A + \boldsymbol{F}_B$$

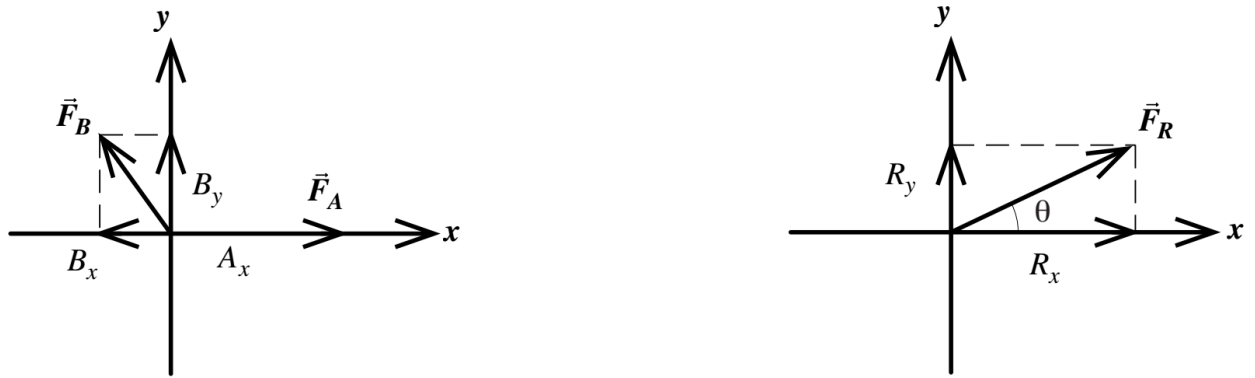*Figure 1: Equilibrant balances the resultant.*

Figure 2: Components

B- Component Method

Two forces are added together by adding the x- and y-components of the forces. First the two forces are broken into their x- and y-components using trigonometry: $\boldsymbol{F_A} = A_x\hat{\imath} + A_y\hat{\jmath}$ and $\boldsymbol{F_B} = B_x\hat{\imath} + B_y\hat{\jmath}$ where A_x is the x-component of vector $\boldsymbol{F_A}$ and $\hat{\imath}$ is the unit vector in the x-direction (see Figure 2). To determine the sum of $\boldsymbol{F_A}$ and $\boldsymbol{F_B}$, the components are added to get the components of the resultant $\boldsymbol{F_R}$:

$$\boldsymbol{F_R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} = R_x\hat{\imath} + R_y\hat{\jmath}.$$

To complete the analysis, the resultant force must be in the form of a magnitude and a direction (angle). So, the components of the resultant (R_x and R_y) must be combined using the Pythagorean Theorem since the components are at right angles to each other:

$$F_R = \sqrt{R_x^2 + R_y^2}$$

The angle is found with the help of trigonometry:

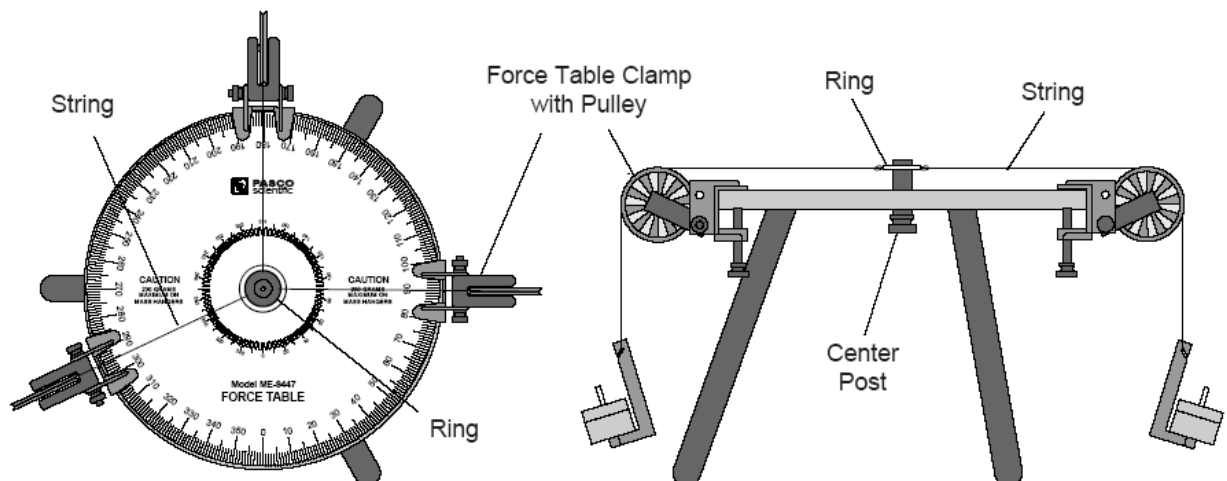$$tan\theta = \frac{R_y}{R_x} \implies \theta = tan^{-1}(\frac{R_y}{R_x})$$

3- SETUP

Figure 3: Force table

1) Assemble the force table as shown in the Figure 3. Use three pulleys (two for the forces that will be added and one for the force that balances the sum of the two forces).

2) Screw the center post up so that it will hold the ring in place when the masses are suspended from the two pulleys.

3) Hang the following masses on two of the pulleys and clamp the pulleys at the given angles: Force A = 50 g at 0° Force B = 100 g at 120°. You <u>may use</u> the coordinate system shown in Figure 4. (Throughout the experiment take the magnitude of the gravitational acceleration as $g = 9.8\,m/s^2$)
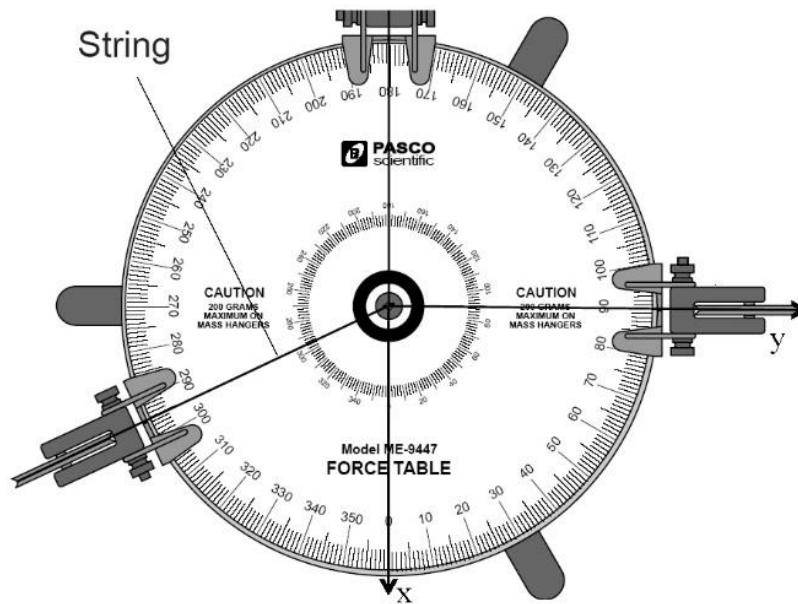

Figure 4: Coordinate system on the force table.

4- ANALYSIS

A) Application of Component Method

Since Force **A** = 50g at 0° Force **B** = 100g at 120° ($g = 9.8\,m/s^2$), to determine theoretically what mass should be suspended from the third pulley and at what angle, first calculate the magnitude and direction of the equilibrant (F_E) by using the "component method". To this end, you should record the vector components of Force **A** (F_A)and Force **B** (F_B) to the Table 1 and compute the magnitude and direction of the equilibrant. (Recall that the equilibrant is exactly opposite in direction to the resultant).

$F_A = - - - - - - - \hat{\imath} + - - - - - - - \hat{\jmath}\ (N)$	$F_R = \sqrt{R_x^2 + R_y^2} = ?$
$F_B = - - - - - - - \hat{\imath} + - - - - - - - \hat{\jmath}\ (N)$	
$F_R = - - - - - - - \hat{\imath} + - - - - - - - \hat{\jmath}\ (N)$	$F_E = ?$
$F_E = - - - - - - - \hat{\imath} + - - - - - - - \hat{\jmath}\ (N)$	$\theta_R = \qquad\qquad\qquad \theta_E =$

Table 1: The components of the force vectors.

B) Application of Experimental Method

- **By trial and error**, find the angle for the third pulley and the mass which must be suspended from it that will balance the forces exerted on the strings by the other two masses. **The third force is called the equilibrant** (F_E) since it is the force which establishes equilibrium. Recall that the equilibrant is the negative of the resultant:

$$-F_E = F_R = F_A + F_B.$$

Ring Method of Finding Equilibrium: The ring should be centered over the post when the system is in equilibrium. Screw the center post down so that it is flush with the top surface of the force table and no longer able to hold the ring in position. Pull the ring slightly to one side and let it go. Check to see that the ring returns to the center. If not, adjust the mass and/or angle of the pulley until the ring always returns to the center when pulled slightly to one side.

- **Record the mass and angle** required for the third pulley (i.e., **the equilibrant**) to put the system into equilibrium in Table 2. **Show all your computations and the forces (F_A, F_B, F_E) on the protractor.**

Mass=$m =$	$F_E =$
$\boldsymbol{F}_E = ------ \hat{\imath} + ------ \hat{\jmath} \ (N)$	$\theta_E =$

Table 2: Equilibrant obtained from the experiment.

Compare the results obtained in Table 1 and Table 2:

Method	*Equilibrant (F_E)*	
	Magnitude	**Direction (θ)**
Experimental: $\boldsymbol{F}_E = ------ \hat{\imath} + ------ \hat{\jmath} \ (N)$	$F_E =$	$\theta_E =$
Component: $\boldsymbol{F}_E = ------ \hat{\imath} + ------ \hat{\jmath} \ (N)$	$F_E =$	$\theta_E =$

Table 3: Comparison of the two methods

Comparing an experimental value to a theoretical value

Percent error is used when comparing an experimental result E with a theoretical value T that is accepted as the "correct" value.

$$\Delta = percent \ (relative) \ error \ = \frac{|T - E|}{T} \times 100\%$$

Fractional or relative uncertainty is used to quantitatively express <u>the precision of a measurement</u>.

$$percent \ (relative) \ uncertainty = \frac{\Delta}{E}$$

Measurement = (measured value ± uncertainty) unit of measurement

1) Determine the percent errors between the experimental value and the theoretical value of the magnitudes and angles. What are the possible two reasons of those errors?
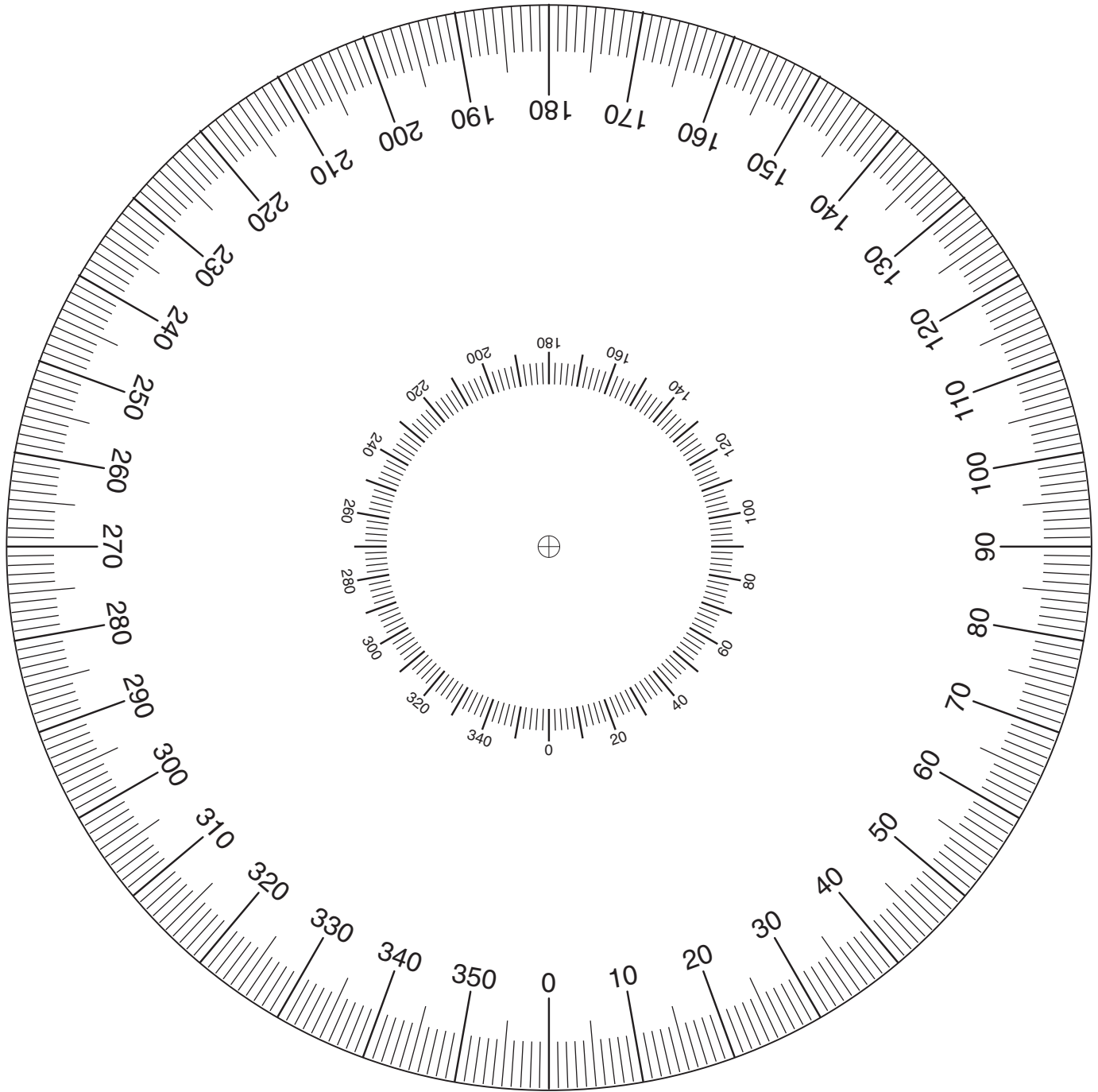
2) Find the precisions of measurements made for the magnitudes and angles.

The protractor on this page is a version of the degree scale on the top surface of the Force Table. It can be duplicated, trimmed, and used as an overlay on the Force Table for drawing and tracing the string positions.
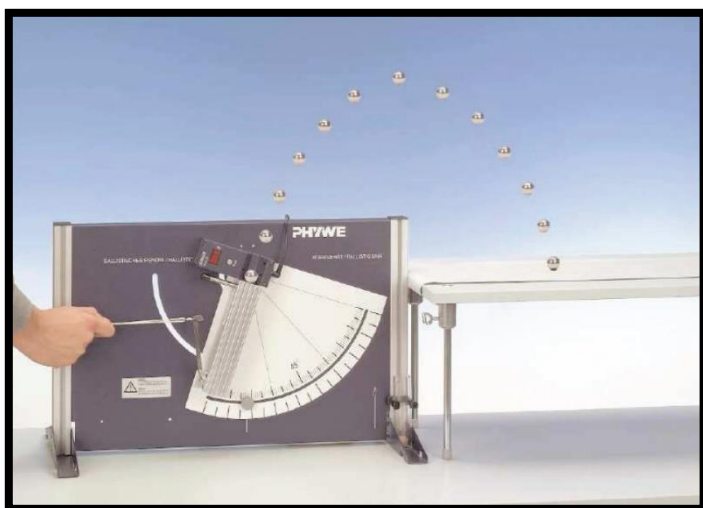
Experiment 2: Projectile Motion

Figure 1: Experimental set-up

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Principle

A ball is fired by a spring at different velocities and at different angles to the horizontal. The relationships between the range, the height of projection, the angle of inclination, and the firing velocity are determined.

Equipment

Recording paper; Ballistic Unit; Speed measuring attachment; Two-tier platform support; Balls; Meter scale; Barrel base expert

Tasks

1. To determine the range as a function of the angle of inclination.

2. To determine the mathematical nature of the projectile motion.

3. To determine the gravitational acceleration g, using the data collected from the experiment.

Theory and evaluation

If a body of mass m moves in a constant gravitational field (gravitational force), the motion lies in a plane (see Fig. 2).
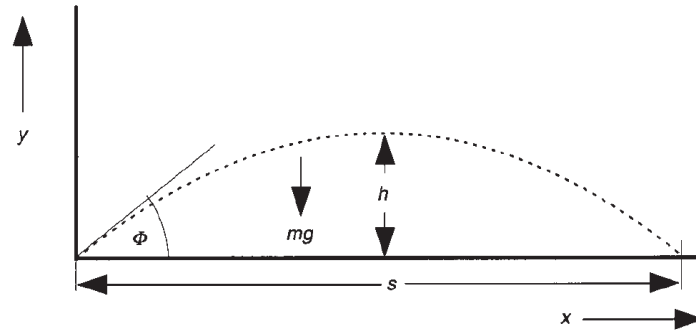

Figure 2: Movement of a mass point under the effect of gravity

If the coordinate system is laid in this plane (Fig. 2) and the equation of motion $m\dfrac{d^2}{dt^2}\boldsymbol{r}(t) = m\boldsymbol{g}$ where $\boldsymbol{r} = x\hat{\imath} + y\hat{\jmath}$ and $\boldsymbol{g} = -g\hat{\jmath}$ is solved with the initial conditions

$$\boldsymbol{r(0)} = 0\hat{\imath} + 0\hat{\jmath},$$
$$\boldsymbol{v_0} = v_0 cos\phi\hat{\imath} + v_0 sin\ \phi\hat{\jmath}$$

to obtain the coordinates as functions of time given by

$$x(t) = (v_0 cos\phi)t,$$
$$y(t) = (v_0 sin\phi)t - \frac{1}{2}gt^2.$$

From these results, one finds the maximum height h and the maximum range s of the projectile as a function of the angle of projection ϕ given by

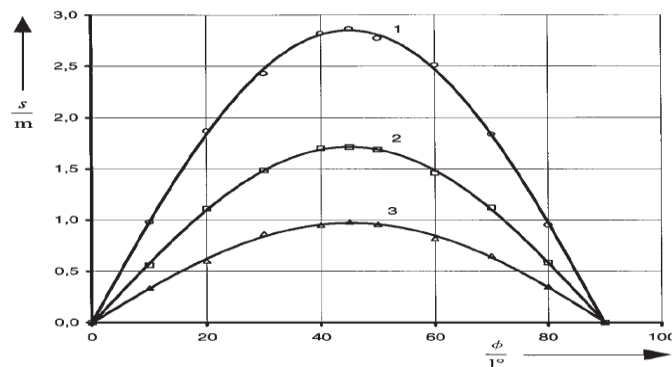$$h = \frac{v_0^2}{2g} sin^2\phi,$$
$$s = \frac{v_0^2}{g} sin\ (2\phi).$$

Figure 3: Maximum range s as a function of the angle of inclination Ø for different initial velocity v_0.

Data in Fig. 3: Curve 1: $v_0 = 5.3\ m/s$, Curve 2: $v_0 = 4.1\ m/s$, Curve 3: $v_0 = 3.1\ m/s$
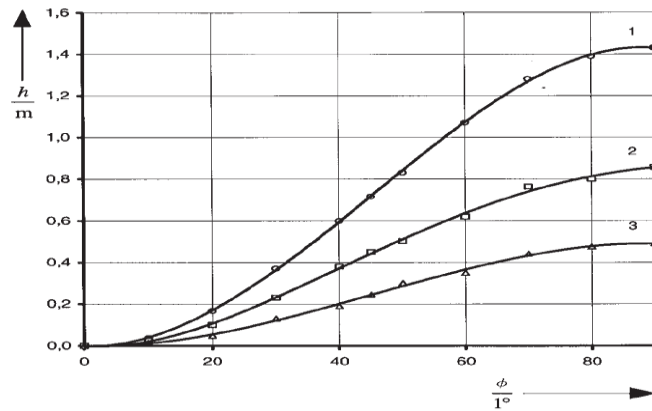
Figure 4: Maximum height of projection h as a function of the angle ϕ of inclination for the initial velocities as in Fig. 1.

The maximum range is reached at an inclination angle of 45° for every initial velocity. Fig. 5 shows the range for an angle of 45° plotted against different initial velocities v_0. By choosing a logarithmic scale, a regression line can be applied to the measured data and used to determine the maximum range for arbitrary initial velocities.
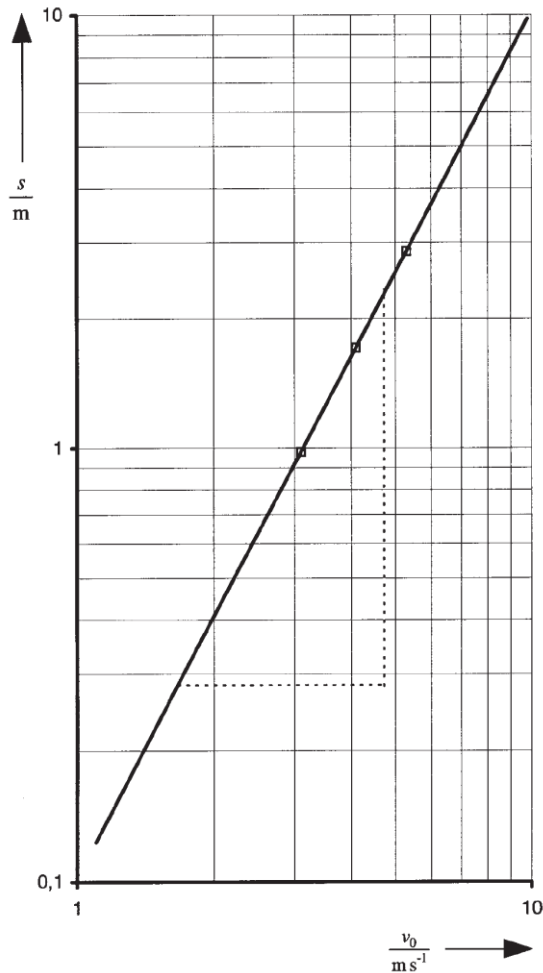

Figure 5: Maximum range as a function of the initial velocity v_0 with a fixed angle of inclination $\emptyset = 45°$.

Note

To ensure an accurate determination of the initial velocity, the time taken for the ball to cover the measuring distance must be considered. Depending on the angle of inclination, the ball already leaves the light barrier with a reduced velocity. If v_{exp} is the experimentally determined initial velocity we obtain as actual initial velocity:

$$v_0 = \sqrt{v_{exp}^2 + g\, d\, sin\phi}$$

where d is the distance between the striker and the center between the light barriers.

Caution:

1) Do not load the launcher while your head or body is in the line of fire.
2) Do not allow anyone to get hit by the ball.
3) Do not perform any launch unless every member in your group is aware and ready.
4) Do not load the launcher for the angled launch unless your instructor is present.
5) You must put the ball in its place, first, and then load the launcher.
6) Do not put your finger inside the launcher.

Procedure (Experimental Method)

Maximum range: Use the ready setup and load the launcher for the minimum initial speed while the initial angle is variable. For each individual angle (in the Table 1) you launch the wooden ball and record its range in the Table 1.

Table 1: Range of the ball versus the initial angle with constant initial velocity v_0.

ϕ	30°	35°	40°	45°	50°	55°	60°
s (m)							
v_0							
$v_0^2 \sin (2\phi)$							

Using the Table 1, sketch the graph of the range s with respect to the initial angle of the projectile motion ϕ in the following graph area.
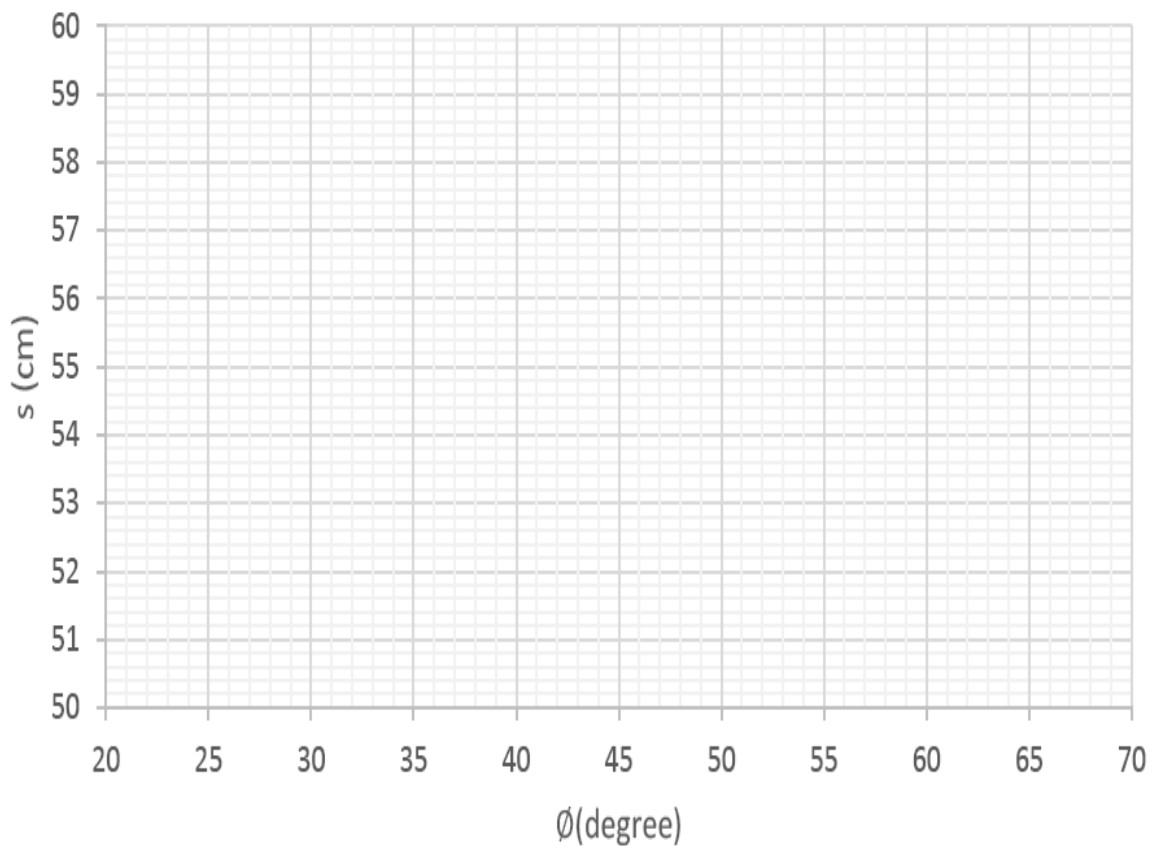

Figure 6: A graph of s versus ϕ.

Measurement of gravitational acceleration

a) Using Table 1, plot the least square line $s(m)$ versus $v_0^2\sin(2\,\phi)$ (m^2/s^2) in the following graph area.

b) Based on the least square line which you have already plotted, and the equation of the range of the projectile motion i.e., $s = \frac{v_0^2}{g}\sin(2\phi)$, **find the experimental value of the gravitational constant g.** This can be done simply by equating the slope of the least square line of the graph and the slope of the equation s with respect to $v_0^2\sin(2\phi)$ which is nothing but $m = \frac{1}{g}$. Note that to get reasonable answer from your analysis, you must use the units of the quantities involved in your calculation correctly.

c) If the theoretical value of the gravitational acceleration is $g = 9.81\ m/s^2$ find the percentage error.

Questions:

1) In the first graph what is the nature of the graph s versus ϕ?

2) What are the possible sources of errors in this experiment? (Show at least two reasons)

3) If one launches the ball vertically i.e., $\phi = 90°$ what will be the maximum height? (Express your answer in terms of v_0 and g)

4) If one launches the ball horizontally i.e., $\phi = 0°$ what will be the range? (Express your answer in terms of v_0 , g and height)

Experiment 3: Newton's Second Law
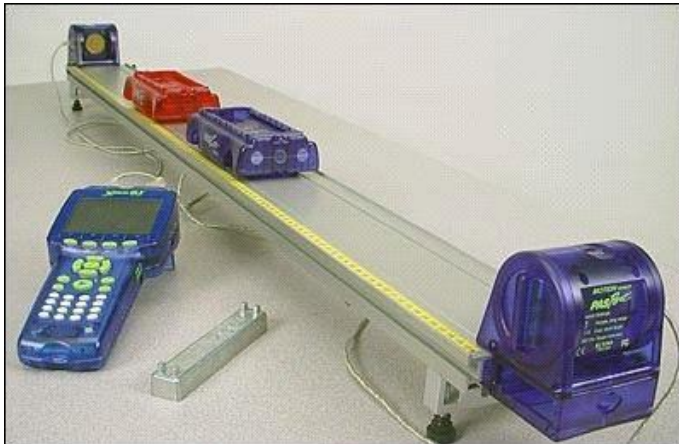
Figure 1: Experimental set-up

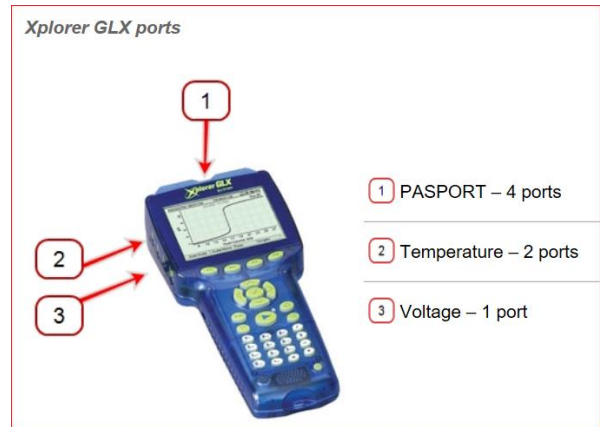| Student Number	
Name & Surname	
Gr	

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Theory

Newton's second law is given by
$$\vec{F}_{net} = \Sigma\vec{F} = m\vec{a}$$
where $\vec{F}_{net}$ is the total force an object, m is the mass, and $\vec{a}$ stands for the acceleration.

The way to test this theory is to apply a force to an object and measure its acceleration. In this experiment, you will learn how to measure the acceleration by using the sonic ranger, getting data (with Xplorer), and draw its associated graph with Excel. To do this, you will use a setup such as the one below.
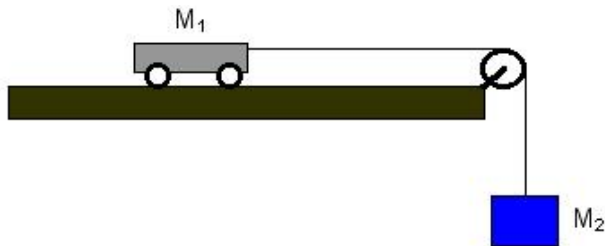
Analyzing this motion may seem difficult, but it is not. By using the **Free Body Diagram** technique, one can get the following equations (all the frictional forces are ignored during the computations):

$$T - M_2 g = -M_2 a$$
$$T = M_1 a$$

By subtracting the above two equations from each other, we obtain

$$M_2 g = (M_1 + M_2)a \qquad (1)$$

In this experiment, you will vary the force, keep the total mass of the system constant, and measure the corresponding acceleration. M_2 will be the sum of the mass of hanger and the various masses hanging on it. The mass reduced from M_2 should be placed on M_1. Thus, the total mass $(M_1 + M_2)$ will always be remained constant.

Procedure

- Start with $0.045 \ kg$ for M_2. This will consist of the hanger (which is $5g = 0.005 \ kg$), two $10g$ and one $20g$ masses: $[M_2 = (5 + 20 + 10 + 10) \times 10^{-3}kg = 0.045kg]$

Masses →

MASS OF HANGER: 5g
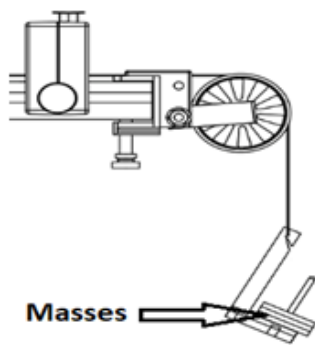
- Add two of the $250g$ bars on the cart having $255g$ tare mass.

MASS OF CART: 255g

- To find the acceleration, you will take the data while the cart is moving towards and away from the sonic ranger. For this aim, you will use the **Xplorer GLX** device. Thus, you will obtain the following data tables of "distance versus time and time square". While doing this, you will reduce the mass on the hanger (i.e., $M_2 \downarrow$) and keep the total mass constant $(M_1 + M_2 = 800g = constant)$.

Fill the following data tables:

Table-1: When $M_2 = 45g$ and $M_1 = 755g$

$d(m)$					
$t(s)$					
$t^2(s^2)$					

Table-2: When $M_2 = 35g$ and $M_1 = 765g$

$d(m)$					
$t(s)$					
$t^2(s^2)$					

Table-3: When $M_2 = 25g$ and $M_1 = 775g$

$d(m)$					
$t(s)$					
$t^2(s^2)$					

Table-4: When $M_2 = 15g$ and $M_1 = 785g$

$d(m)$					
$t(s)$					
$t^2(s^2)$					

and draw the graph by Excel.

- For each data table, draw the graph of d (m) versus $t^2(s^2)$ by using Excel program.
- Find the slope of the least square line of each $d(m)$ versus $t^2(s^2)$ graph and calculate the associated accelerations. (Hint: Recall that $d = \frac{1}{2}at^2$).
- Copy the obtained accelerations to the following table:

Table-5:

$M_2(kg)$	0.045	0.035	0.025	0.015
$a(m/s^2)$				

Finding Gravitational Acceleration

- By using Excel plot the least square line of $a(m/s^2)$ versus $M_2(kg)$ graph.

- Find the slope of the least square line of $a(m/s^2)$ versus $M_2(kg)$ graph.

- Use Eq. (1) to find the experimental value of the magnitude of gravitational acceleration g:

  $g_{exp} = \ldots\ldots\ldots\ldots$

- If the theoretical value of $g_{th} = 9.81(\frac{m}{s^2})$, find the percentage error and the precisions of measurements made for the magnitude of the gravitational acceleration.

  Recall that "$\Delta = percent\ (relative)error = \frac{|T - E|}{T} \times 100$ & $Percent\ (relative)\ uncertainty = \frac{\Delta}{E}$"

Questions

1. What physical property does the slope of a Force v Acceleration graph represent?

2. In the absence of an external force, a moving object will
    a) move with constant velocity.
    b) stop immediately.
    c) slow down and eventually come to a stop.
    d) go faster and faster.

3. You are standing in a moving bus, facing forward, and you suddenly fall forward as the bus comes to an immediate stop. What force caused you to fall forward?
    a) Gravity.
    b) Normal force due to your contact with the floor of the bus.
    c) Force due to friction between you and the floor of the bus.
    d) There is not a force leading to your fall.

4. A $20N$ weight and a $5N$ weight are dropped simultaneously from the same height. Ignore air resistance. Compare their accelerations.
    a) The 20 N weight accelerates faster because it is heavier.
    b) The 20 N weight accelerates faster because it has more inertia.
    c) The 5.0 N weight accelerates faster because it has a smaller mass.
    d) They both accelerate at the same rate because they have the same weight to mass ratio.

5. Who has a greater weight to mass ratio, a person weighing 400 N or a person weighing 600 N?
    a) the person weighing 400 N
    b) the person weighing 600 N
    c) Neither; their ratios are the same.
    d) The question can't be answered; not enough information is given.

Experiment 4: Conservation of Energy

Student Number	
Name & Surname	
Gr	

Equipment: Roller Coaster Complete System; Photogate Heads; Smart Timer

Introduction

A car is started from rest on a variety of shapes of tracks and the speeds of the car at various points along the track are measured using a photogate connected to a Smart Timer. The potential energy is calculated from the measured height and the kinetic energy is calculated from the speed. The total energy is calculated for two points on the track and compared.

The height from which the car must be released from rest to just make it over the loop can be predicted from conservation of energy and the centripetal acceleration. Then the prediction can be tested on the real roller coaster. Also, if the car is released from the top of the hill so it easily makes it over the top of the loop, the speed of the car can be measured at the top of the loop and the centripetal acceleration as well as the apparent weight (normal force) on the car can be calculated.

Theory

The total energy (E) of the car is equal to its kinetic energy ($K = \dfrac{1}{2}mv^2$) and its potential energy ($U = mgh$), where m is the mass of the car and v is the speed of the car. g is the acceleration due to gravity and h is the height of the car above the position, where the potential energy is defined to be zero. If the friction is ignored, the total energy of the car does not change. The Law of Conservation of Energy is stated as

$$E = constant \implies K_{initial} + U_{initial} = K_{final} + U_{final}$$
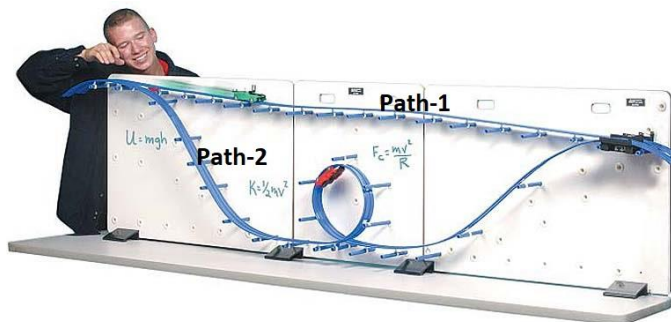
1

A-Step Procedure

Figure 1: Step Configuration

1. Follow the **Path-1** seen in Figure 1.

2. Place the car (**without extra mass attached on it**) to the starting point at the leftmost of the track and measure the height (h_i) of the car from the surface of the table. **Note that the center of mass of the car is approximately at the slot where the flag is inserted.**
(Car's tare mass is $m = 42.7g$).
$h_i =$

3. Place the car at the top and release it from rest. Use the photogate and Smart Timer (**set on the <u>Speed: One Gate Mode</u>**) to measure the speed of the cart at the bottom of the step.

4. Calculate the initial total energy of the car.
$E_i =$

5. Calculate the final total energy of the car.

$E_f =$

6. How much energy is lost? Where does it go?

$E_{lost} =$

7. Calculate the percent of **total** energy lost.

$\% \, Lost = \% \, \dfrac{E_{lost}}{E_i} \times 100$

8. **Attach the 50g mass to the car and repeat steps 2 through 8 above.**

$E_i =$	$E_{lost} =$
$E_f =$	$\%Lost =$

Questions

1. How does increasing the mass of the car change the total energy?

2. How does increasing the mass of the car change the speed of the car at the bottom?

3. Does the car lose a greater percentage of its energy when it has the extra mass or not?
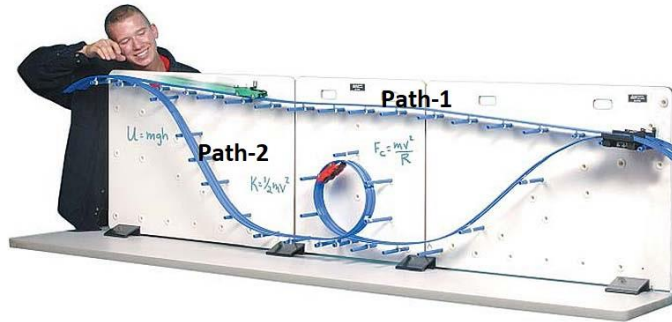
2

B-Loop Procedure

Figure 2: Loop Configuration

1. Follow the **Path-2** seen in Figure 2.

2. Place the car (**with attached extra mass 50g**) to the starting point at the leftmost of the track. Recall that you have already measured the height (h_i) of the car from the surface of the table in A-2 part. ($m = 92.7g$)
$h_i =$

3. Calculate the initial total energy of the car.
$E_i =$

4. Calculate the final total energy of the car.
$E_f =$

5. How much energy is lost? Where does it go?
$E_{lost} =$

6. Calculate the percent of **total** energy lost.

$$\% \, Lost = \% \, \frac{E_{lost}}{E_i} \times 100$$

7. By using the speed of the car at the top of the loop and the centripetal acceleration formula, **compute the magnitude of the apparent weight** (**normal force**) on the car.

Question

1. Comparing the result of section B with the result obtained in section-A, for the car with 50g extra mass, which car does have the greater speed at the right end of the track since they have almost same height difference? How does energy conservation explain your answer?
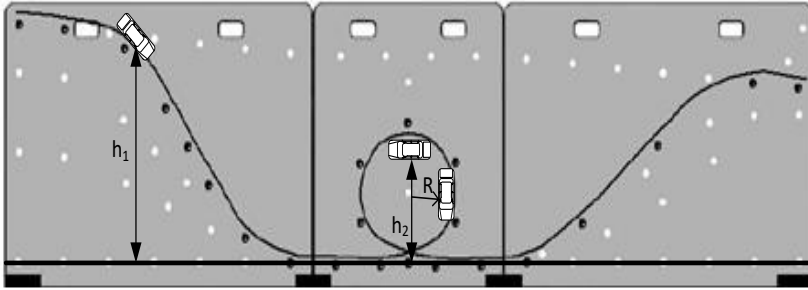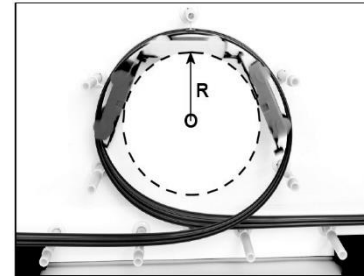
3

C- Critical Height

Figure 3

Figure 4

1. Follow the **Path** seen in Figure 3.

2. Measure from the center of the loop to the center of mass of the car (**without extra mass**) at the top of the loop (see Figure 4), and obtain $R =$?

$R =$

3. Measure the distance from the center of mass of the car at the top of the loop to the table ($h_2 =$?).

$h_2 =$

4. Using Conservation of Energy, compute the **critical height** ($h_1 =$?) from which the car can be released on the left end of the track, so that the car **will just lose its contact** with the surface at the top of the loop. At that moment, the critical speed is $v \equiv v_c = \sqrt{Rg}$.

$$(Hint: mgh_1 \cong \frac{1}{2}mv^2 + mgh_2)$$

5. Place the center of mass of the car at the h_1 position and release it from rest.

Question

Does the car make the loop over? Explain why or why not?

Experiment 5: Conservation of Linear Momentum and Kinetic Energy

Eastern Mediterranean University
"Virtue, Knowledge, Advancement"

Student Number	
Name & Surname	
Gr	

PURPOSE

In a collision process on an air track to check that linear momentum and total kinetic energy are conserved.

APPARATUS

Air track, gliders with flags, electronic timers mounted to photogates.

Figure 5.1

THEORY

On an air track, in principle, the friction is reduced to zero. For this reason, it becomes very suitable to make collision experiments on an air track. For this purpose, we use two gliders with variable masses on the air track and two photogates to record electronically the time intervals. Each 5.1 has a flag on it with a length of $l = 10\ cm$. (see Fig. (5.1)). Prior to the collision we denote the speeds of objects (the gliders) by v_1 (with mass M_1) and v_2 (with mass M_2). After the collision these become v_1' and v_2', respectively. Accordingly, the conservation principle of linear momentum is expressed by

$$M_1\vec{v}_1 + M_2\vec{v}_2 = -M_1\vec{v}_1' + M_2\vec{v}_2'.$$

Since the air track is a one-dimensional object the process takes place in one-dimension only, say the x-axis. Depending on the directions of velocities, the velocities can at most be $\pm$ in sign. Similarly, the conservation principle for the total kinetic energy in an Elastic Collision can be expressed by

$$\frac{1}{2}M_1{v_1}^2 + \frac{1}{2}M_2{v_2}^2 = \frac{1}{2}M_1{v'_1}^2 + \frac{1}{2}M_2{v'_2}^2.$$ 5.2

Let us remind that each kinetic energy term is automatically positive. In order to make sure that no potential energy due to gravity is involved the air track must be perfectly horizontal. This must be checked in the beginning before the experiment is started. What we aim in this experiment essentially is to check the validity of Eq. (5.1) and Eq. (5.2).

The most important part of the experiment is to find the speeds correctly. For this purpose, we use photogates that electronically record time passage for each glider. If the time is recorded by t_1 and the length of the flag on the glider is l_1 then, from the relation

$$v_1 = \frac{l_1}{t_1},$$ 5.3

we can find the speed v_1 and similarly the other speeds. In order to make the experiment simpler, so that we can conduct better measurements we assume throughout the experiment that $v_2 = 0$, that is one of the objects (M_2) is at rest initially. This casts our equations of linear momentum and kinetic energy conservations into the forms

$$M_1 v_1 = -M_1 v'_1 + M_2 v'_2,$$ 5.4

$$\frac{1}{2}M_1{v_1}^2 = \frac{1}{2}M_1{v'_1}^2 + \frac{1}{2}M_2{v'_2}^2,$$ 5.5

so that we have to find only three speeds instead of four.

EXPERIMENTAL PROCEDURE

First you have to check that the air track is perfectly horizontal with zero inclination angle. Then, put the masses provided in the table below, on each glider. Make the arrangement as in Fig. (5.1).

2

One of the gliders (M_2) at rest is between the two photogates while the other one (M_1) lies outside the photogates. Before we move M_1 with initial speed v_1, we turn on the pump of the air track (to maximum level) so that friction becomes zero.

As M_1 passes through one of the photogates its time will be recorded. Fill the following table knowing that the tare masses of the gliders are same and equal to $m_{glider} = 0.210 kg$. The two gliders will collide and they will pass through different photogates. In this way their times (t_1, t_1' and t_2') will be recorded and speeds will be found from Eq. (5.3).

RAW DATA

	$t_1(s)$	$t_1'(s)$	$t_2'(s)$	$v_1 = \dfrac{0.10}{t_1}$	$v'_1 = \dfrac{0.10}{t'_1}$	v_2	$v'_2 = \dfrac{0.10}{t'_2}$	$M_1(kg)$	$M_2(kg)$
1						0		0.210	0.310
2						0		0.210	0.410
3						0		0.230	0.310
4						0		0.230	0.410

DATA ANALYSIS

We shall be knowing now, $v_1, v_1',$ and v_2' to be substituted into Eqs. (5.4) and (5.5). If Eq. (5.4) is satisfied this will mean that in the process of collision linear momentum is conserved. Similarly, Eq. (5.5) will tell us that the total kinetic energy is conserved in the process.

In such an experiment we must admit that a few percent error between the left- and right-hand sides of Eqs. (5.4) and (5.5) are tolerable. The students must be able to explain about the possible numerical differences in those equations. For example, is the collision exactly elastic? Is there some energy loss taking place in the collision process? How?

CAUTION

If the difference between the left- and right-hand sides of Eqs. (5.4) and (5.5) are big, that means the data taken is not recorded correctly. So, please repeat the experiment and reduce your errors as

much as possible. For example, if M_1 comes to a stop after collision you should increase its initial speed so that it will gain enough speed to pass through the photogate after the collision.

QUESTIONS

1) What are the differences between elastic, inelastic and completely inelastic collisions?

2) Can you give an example for each different case?